1301. Use the exact values $\sin 30° = {}^1\!/_2$, $\cos 30° = \sqrt{3}/2$. Then use $\tan x \equiv \frac{\sin x}{\cos x}$.

1302. (a) Evaluate $\left.\frac{dE}{dt}\right|_{t=0}$
 (b) A quantity q is increasing when $\frac{dq}{dt} > 0$.
  i. Solve the relevant inequality.
  ii. Differentiate and solve the inequality.

1303. "Mutually exclusive" means that A and B never happen together; "independent" means that the occurrence of one does not affect the probability of the other.

1304. The two parts of the question are the same: $\ln x$ means "logarithm with base $e \approx 2.718$".

1305. Call the distance r, and use the sine area formula for a triangle on one of the sectors of the octagon.

1306. The quadratic has roots at $x = 2$ and $x = 4$, and the area beneath the graph between those limits is positive.

1307. (a) Use the volume of a prism $V = lA$.
 (b) Use *suvat*, converting to metres.
 (c) Use NII with the object modelled as "the water in the pipe".

1308. Sketch the lines carefully, and consider lines of symmetry.

1309. (a) Consider the definition of the mean

$$\bar{x} = \frac{\sum x}{n}.$$

 (b) Consider the definition of the variance

$$s_x^2 = \frac{\sum (x_i - \bar{x})^2}{n}.$$

1310. Neither is true; look for specific counterexamples.

1311. Divide the equilateral faces into two right-angled triangles each.

1312. Split a sector, subtending 36° at the centre, into two right-angled triangles. By definition,

$$\operatorname{cosec} 18° = \frac{1}{\sin 18°}.$$

Each reciprocal trig function can be identified by its third letter, e.g. cosec is the reciprocal of sin.

1313. In e.g. the friction experienced by a car braking to a halt, consider the *two* objects on which the friction acts.

1314. Carry out the definite integrals on each side. Thus form and solve an equation in k.

1315. The function $f(x) = 2^x$ has range $(0, \infty)$. The other two functions can then be thought of as transformations of this function, with either the inputs or outputs negated.

1316. In a monic parabola, the coefficient of x^2 is 1. Hence, this new parabola must a reflection of the old one in the y axis. So, replace x by $-x$.

1317. One of these is true; two are false.

1318. You can solve this problem without calculation. Consider the degree of the equations involved, i.e. quartics and cubics. Every cubic equation must have a real root.

1319. Consider $x = a$ and $x = b$.

1320. Rearrange each equation to the form $y = mx + c$, and then set up an equation $m_1 m_2 = -1$.

1321. The sets on the left are
 (a) positive integers,
 (b) non-positive integers,
 (c) integers whose magnitude is 2 or more.

1322. Express the AP generically as $u_n = a + (n-1)d$, and integrate $f'(x) = k$.

1323. Consider the equation as a quadratic in t^3. Making t the subject is the same as solving for t.

1324. The error is in "..., the null hypothesis *is* false."

1325. Draw a force diagram, and consider the horizontal equilibrium (or lack of it).

1326. Use $\sin^2 x + \cos^2 x \equiv 1$, and then the discriminant $\Delta = b^2 - 4ac$.

1327. Rewrite $\frac{\pi i}{20}$ as $\frac{2\pi i}{40}$, and consider the fact that 2π radians is one full rotation about the centre.

1328. Sketch the curves to get your bearings, considering the fact that the curves are reflections in the line $y = x$.

1329. For a graph $y = h(x)$, consider the nature and location of the stationary point at $x = 3$.

1330. Use log laws to put the equation in the form $\ln p = \ln q$, then exponentiate both sides.

1331. Sketch the scenario. Use $A_\triangle = \frac{1}{2}bh$.

1332. Give all possible functions $g'(x)$, using the fact that $g'(x)$ has roots at $x = \pm 2$, then integrate.

1333. Rearrange to $|x| > 1 - x$, and sketch $y = |x|$ and $y = 1 - x$.

1334. Begin by moving the $\tan y$ term to the RHS.

1335. The curve is an ellipse.

1336. The effect of the chain rule has been neglected.

1337. Split the velocity into components $u_x = 70\cos 15$ and $u_y = 70\sin 15$, then set up a vertical *suvat* with $s_y = 0$. Calculate the time taken and use it to find s_x.

1338. Multiply both sides of the equation by $(1 + x)^2$, then simplify.

1339. You know that $a(1 - r)^2 > 0$. Begin with this inequality and rearrange to the required result.

1340. (a) Scale $\frac{4}{15}$ of the sample up, leaving the other $\frac{11}{15}$ unscaled.
   (b) Consider sampling variation, of the four out of the fifteen.

1341. Use log rules, with $\ln x$ defined as $\log_e x$.

1342. Find the equation with $y - y_1 = m(x - x_1)$, and solve for the x intercept.

1343. Consider the relevant gradient triangles. In each case, the denominator is the difference between the x coordinates of the endpoints of the chord.

1344. Equate the differences $u_2 - u_1 = u_3 - u_2$.

1345. Assume that every pupil has a $\frac{7}{365}$ probability of having a birthday in a given week.

1346. Multiply up by the denominators.

1347. Solve $2x^3 - 5x^2 - 21x + 36 = 0$ on your calculator, then reverse engineer the factorisation.

1348. (a) fg means "perform g, then f",
   (b) The sequence is periodic; find the period.

1349. Draw a possibility space as a 4×6 grid.

1350. Begin with $\sin^2\theta + \cos^2\theta \equiv 1$.

1351. (a) There are nine forces in total.
   (b) The minimum driving force occurs when the acceleration is zero. Resolve up the slope for the entire system.
   (c) Use NII for the caravan alone.

1352. This is a disguised quadratic, in $\sqrt{x}$.

1353. (a) The behaviours are different above and below the line $y = q$.
   (b) The behaviours are different to the left and right of the line $x = p$.

1354. Call the side length of the square/octagon 1, and calculate the area of the octagon. This area is then the possibility space.

1355. Substitute $y = x^2$ into (c) and see if it holds.

1356. Simplify each side separately as an expression, to reach $(n + 1)^2$.

1357. Multiply out the top of the integrand, then split the fraction up.

1358. Find the equation of the perpendicular bisector of AB and of AC, and solve to find their intersection. This is the centre of the circle. Then calculate the radius using Pythagoras.

1359. Consider the numerator as a difference of two squares.

1360. Distribute the differential operator $\frac{d}{dx}$ over the bracket, i.e. apply it to each term individually. Rearrange to make $\frac{dy}{dx}$ the subject.

1361. In both parts, consider the two competitors and rope as a single object, and use integration on $F = ma$.
   (a) Find the maximum displacement attained (in the direction in which competitor 1 is pulling) by setting the velocity to zero.
   (b) Set the displacement to -1 (or 1 depending on how you've set the problem up) and solve.

1362. Use $|a| = |b| \iff a = \pm b$.

1363. The first is false. Find a counterexample.

1364. Draw a possibility space.

1365. Using the factor theorem, $(15x - 4)$ is a factor.

1366. This can be done algebraically, using Pythagoras and solving for the parameter t, or geometrically, by sketching.

1367. The differential operator $\frac{d}{dt}$ can be distributed over addition, i.e. you can apply it to each term in the bracket. The first term gives $4\frac{dx}{dt}$.

1368. This is a simpler question than it looks. Place the first point somewhere. Then each subsequent point has a 50% chance of being separated from it.

1369. This is the factor theorem.

1370. Give the box mass m, and draw a force diagram for it.

1371. (a) Draw a circle of radius 5 at one end of a side of length 10. Add the angle 25° at the other end. You should see two possible locations for the third vertex.
   (b) Either consider, in each case, the particular line of algebra at which two answers appear, or else simply do the calculations!

1372. This can be viewed either as a geometric series, or, equivalently, as a recurring decimal.

1373. Using $\mathbb{E}(X) = np$, the expectation is $1.\dot{6}$. So, check 1 and 2.

1374. (a) Consider the *nature* of any stationary points.
   (b) Use the fact that, in a region in which $y = \text{h}(x)$ is increasing (or decreasing) everywhere, it can have at most one x intercept.

1375. Find the segment area in terms of k: calculate the sector area and subtract the triangle. The fact that k is rational allows you to equate coefficients.

1376. Sketch $y = \sqrt[3]{x}$ first, considering it as a reflection of $y = x^3$ in the line $y = x$. Then perform two output transformations.

1377. Eliminate z from equations 1 and 2, and then from equations 1 and 3. Solve the resulting equations in x and y.

1378. You can't use the factor theorem here (at least not over the real numbers). Attempt the factorisation explicitly, writing

$$4x^3 - 12x^2 + 18 \equiv (x^2 + 1)(Ax + B).$$

Find a contradiction.

1379. Converting to continuous inputs n, use calculus or complete the square to find the minimum point, then consider the integers either side of it.

1380. Consider the $+c$ in integration.

1381. (a) Set up force diagrams, find the acceleration, then use *suvat*.
   (b) Use *suvat* with $a = -g$.

1382. Consider the equation formed were you to solve to find intersections of the cubic and the relevant straight line.

1383. The Newton-Raphson iteration is

$$x_{n+1} = x_n - \frac{\text{f}(x_n)}{\text{f}'(x_n)}.$$

Put the RHS over a common denominator.

1384. Consider the relevant graphs. Any linear factor must correspond to a root. Do cubic, quartic or quintic graphs necessarily have at least one root?

1385. Solve simultaneously to find **a** and **b** explicitly. **i** and **j** are the standard perpendicular unit vectors in the x and y directions.

1386. Consider the effect on $\sin^2\theta + \cos^2\theta \equiv 1$ when replacing the relevant inputs.

1387. Rewrite the original log statement as an index statement, then cube root both sides.

1388. Consider the number of successful outcomes in each case. You can use a conditioning approach or a combinatorial approach. In the latter, there are ${}^6\text{C}_3 = 20$ outcomes in the possibility space.

1389. Write the sum out explicitly, and multiply up by the denominators.

1390. The first circle passes through the origin.

1391. Put $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ over a denominator of 12.

1392. Split each shaded section up into a pair of right-angled isosceles triangles.

1393. Sketch the curves.

1394. Consider whether every element of the domain maps to an element of the codomain.

1395. Express the fact "f″ is linear" as $\text{f}''(x) = ax + b$, and integrate twice. Consider separately the cases where $a = 0$ or $a \neq 0$.

1396. (a) Express the surface area in terms of a and b, and then use the volume formula to substitute for b.

   (b) Differentiate to find the values of a at which A is stationary relative to a.

1397. (a) Set $y = 0$ and solve.

   (b) Differentiate with respect to t.

   (c) Substitute for y and for $\frac{dx}{dt}$, simplifying in terms of t. Then perform the definite integral over t.

1398. This is not possible. Find the reason.

1399. Start with the LHS, expressing x as a^*.

1400. (a) Condition on passing the first test.

   (b) Restrict the possibility space to outcomes in which the candidate is not accepted.

   (c) This is now a binomial problem.

——— End of 14th Hundred ———